

# HEART RATE VARIABILITY ANALYSIS THROUGH MULTIRESOLUTION ORTHOGONAL WAVELET TRANSFORM

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**Abstract:** Heart rate fluctuations contain important diagnostic information about a number of biological control and feedback subsystems. In the present study we show that the multiresolution decomposition through orthogonal wavelet transform is able to reveal short-time as well as long-term heart rhythm variations. We demonstrate that return maps and autocorrelation of the low/high frequency regions of the heart rate variability can be very useful for understanding the underlying chaotic and deterministic control patterns.

**Keywords:** heart rate variability, orthogonal wavelet transform, multiresolution decomposition, return maps, chaos

## **Introduction**

Noninvasive methods for assessment of cardiac rhythm control are indispensable for prevention of sudden cardiac death, for early detection of diabetic neuropathy in children, for postsurgical follow-ups and mass-screenings. Heart rate variability (HRV) is a sensitive indicator for subtle status changes of various physiological factors [1]. The cardiovascular control is initiated by the corresponding hypothalamic centers. However, the heart rhythm is controlled by the response of the autonomic nervous system to a number of fluctuations in the human body homeostasis due to functional demands and changing environmental conditions, by cardiovascular reflexes (baroreceptor, chemoreceptor and cardiac reflexes, respiratory sinus arrhythmia), by intrinsic cardiac factors (cardiac rhythmicity) and by hormonal influences.

True sinus rate for HRV studies should be derived from the ECG through the onsets of the P waves, but the R waves are much more reliably identified by computer programs and R-R intervals are generally treated as a close approximation of the P-P intervals. Using standard time-frequency distributions (Fourier Transform) [2] or time-dependent techniques (autoregressive modeling) for computing the HRV spectra is considered to give a very useful picture of the instantaneous interaction between the two autonomic nervous systems, which regulate (directly or indirectly) major body functions. In general, stimulation of the sympathetic nervous system is associated with increased heart rate and strength of contraction, as well as with decreased HRV, while parasympathetic stimulation of the vagus nerve to the heart causes an opposite balance shift. Although the exact mechanisms of this interplay are not yet very well understood and not all studies agree on the details, parasympathetic activity is regarded as more correlated with High Frequency (HF) components of spectra, but both sympathetic and parasympathetic tonuses seem to determine the Low Frequency (LF) components of the HRV [3].

An established diagnostic method for neurocardiac control assessment is to compute the LF:HF ratio from the HRV spectra. The essential frequency bands in these spectra are well known [2]: 0-0.0167 Hz - dominated by physical activity and postural changes; 0.0167-0.05 Hz: Very LF (vasomotor and renin-angiotensin control systems - the thermoregulatory frequency is approx. 0.03 Hz); 0.05 Hz - 0.15 Hz: LF (oscillations of the baroreflex system - the Traube-Hering-Meyer frequency is approx. 0.1 Hz) and

0.15-0.4 Hz: HF (the respiratory arrhythmia frequency is 0.25 Hz for 15 breath/min; for normal free-breathing - 0.15-0.4 Hz).

However, these prevalently used spectral estimation techniques are bound to make a priori assumptions that the signal is at least quasi-stationary (or that it follows such a model), which is unfortunately often not the case in the clinical practice. Another disadvantage of such methods is that, while they may have a good frequency resolution for a particular study, usually due to the uncertainty principle and fixed window size their time resolution is insufficient for precise long-term evaluations.

Under pathological conditions switching between several underlying HRV patterns is not unusual in long-term ECG recordings. These patterns, as well as their timing and order contain important diagnostic information. Even normal short-time physical and mental load conditions could make the assumption for signal stationarity inappropriate due to dynamic internal balance shifts. Any transient or complex non-deterministic signal patterns could be systematically misinterpreted or ignored.

Clearly, another decomposition method is required for a more successful assessment in time of the highly informative low- (LF) and high-frequency (HF) components of the HRV. The fine temporal fluctuations of these components and the physiological interpretation of the changes is still largely not well understood and need more investigation. A number of efficient time-frequency decomposition methods are currently used in biomedical applications, but non-redundancy, computational efficiency, constant-Q processing and multiresolution properties point assertively at the discrete wavelet transform (DWT) as a good possible candidate for the two specific goals of the present study:

- (1) to follow up precisely the trends in the autonomic control status during a long-term session;
- (2) to extract significant singularities in the cardiac sinus rhythm, in order to associate them with normal or pathological terms.

## Methods

### A. The Continuous Wavelet Transform

The wavelet transform (WT), presented by J. Morlet et al. [5], is an “affine” distribution, invariant to shift and dilatation. In general, the WT of a function  $f(x)$  at scale  $a$ , with respect to a mother wavelet  $\psi(x)$ , is the convolution:

$$W_a f(x) = f * y_a(x),$$

where  $\psi_a(x) = a^{-\frac{1}{2}} \psi(a^{-1}x)$ .

The admissibility condition for  $\psi(x)$  reads (essentially):

$$\int \psi(x) dx \approx 0,$$

which explains why Morlet called these functions “wavelets”, meaning “little waves”.

More explicitly, the continuous wavelet transform (CWT) is the modified convolution:

$$W(a, b) = |a|^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(x) \psi\left(\frac{x-b}{a}\right) dx.$$

The scale  $a$  and translation  $b$  parameters can vary in a continuous ( $a, b \in \mathfrak{R}, a \neq 0$ ) or discrete way ( $a = a_0^m$ ;  $b = nb_0 a_0^m$ , where  $a_0 = 2$  and  $b_0 = 1$ ). Increasing the scale  $a$  produces a family of dilated wavelets with a common origin (the mother wavelet), while  $b$  shifts these wavelets within the data set. The energy normalization term of the CWT allows equalizing the energies of the mother and all child wavelets (resolution of identity property). When the input signal is projected onto such a dilated and translated set of functions, the obtained two-dimensional surface of wavelet coefficients reflects essentially the frequency content of the signal, changing in time.

### B. The Discrete Wavelet Transform

The multiresolution wavelet representation [6] ( $A_{2^{-j},l} f, (D_{2^{-j},l} f)_{-J \leq j \leq -1}$ ) decomposes a sequence  $f(x)$  of length  $N = 2^J$  into  $J$  multiresolution levels of approximated  $A_{2^{-j},l} f$  and detail  $D_{2^{-j},l} f$  signals at a time point  $l$

$$A_{2^{-j-1},l} f = \sqrt{2} \sum_k \tilde{h}_{k-2l} A_{2^{-j},k} f$$

$$D_{2^{-j-1},l} f = \sqrt{2} \sum_k \tilde{g}_{k-2l} A_{2^{-j},k} f,$$

where  $\tilde{g}_k = (-1)^k \tilde{h}_{-k+1}$ .

At each level  $j$  the discrete quadrature mirror filters (QMF)  $H$  (low-pass) and  $G$  (band-pass), corresponding to particular scale and wavelet functions, are convolved with the approximated and subsampled input signal from the previous pyramid level. We would like to recall that QMFs are discrete filters  $H = (h_n)_{n \in \mathbb{Z}}$ , whose transfer function satisfy the condition [7]

$$|H(e^{-iv})|^2 + |H(-e^{-iv})|^2 = 1,$$

so that a perfect reconstruction of the analyzed signal can be achieved.

The subband coding algorithm produces coarse and detailed output signals at the currently analyzed octave band and the wavelet expansion  $D_{2^{-j},l} f$  splits the frequency space into non-overlapping dyadic blocks

$$[-2^{-j+1} \pi, -2^{-j} \pi] \cup [2^{-j} \pi, 2^{-j+1} \pi]$$

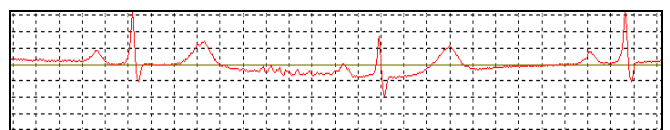
with a constant size on the logarithmic scale.

For high frequencies the time resolution grid of the WT becomes better and the frequency resolution - worse, while for low frequencies of  $f$  the opposite assertion is valid [9].

### C. HRV Analysis

We analyzed 20 long-term (30 min.) ECG recordings (Fig.1) from the AHA database through wave recognition techniques described earlier [4]. After exclusion of all ectopic beats the original HRV time series (Fig.2) was interpolated to a new equidistant HRV time series at sampling rate  $w_s = 5$  Hz.

Fig.1. ECG data segment, extracted from a long-term recording of a normal patient. Vertical Grid is 100 ms;



series duration is 3 min.

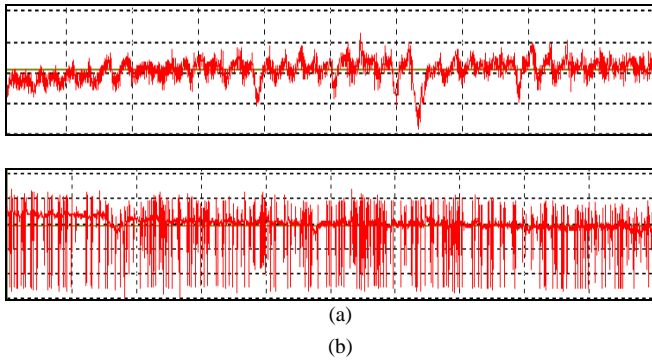


Fig.2. Original R-R interval variability time series (a) a normal subject with typical frequency fractions in the FFT heart rate variability spectrum; (b) a subject with pathological heart rate variability. Data is obtained following adaptive wave recognition procedures, described elsewhere [4]. After an equidistant time interpolation this series serves as an input to the Short-Time Fourier and to the orthogonal Wavelet Transform. Vertical Grid is 100 ms, while series overall duration is 30 min.

We tested the performance of a number of wavelets (Daubechies, Meyer, Coifman, Battle-Lemarie) with various filter lengths. In our current study we performed most of the tests using the orthogonal Daubechies 4-tap filter for its advantageous feature extraction, compact support and high computational efficiency.

Out of 13 available frequency bands of the dyadic DWT decomposition we used the following 7 physiologically relevant ones:

LF: 0.00488-0.00976 Hz, 0.00976-0.01953 Hz,  
 0.01953-0.03906 Hz, 0.03906-0.07812 Hz,  
 0.07812-0.15625 Hz;  
 HF: 0.15625 - 0.3125 Hz, 0.3125 - 0.625 Hz.

We analyzed the square moduli of the dyadic wavelet coefficients  $D_{2^{-j}}f$ , setting to zero the bottom 3 % of them to avoid low noise and studied in detail the time courses of the LF,

$$d_{LF}(l) = \sum_j^{LF} (D_{2^{-j},l}f)^2,$$

HF, LF+HF band energies and the LF/HF band energy ratio of the wavelet transformed HRV.

The constant-Q filtering property of the DWT for our HRV data sets was accounted by the quality factor

$$Q = \frac{1}{90.5} \quad (Q = \frac{\text{center frequency}}{\text{bandwidth}}).$$

As expected, DWT energy plots had only a distant resemblance to their corresponding STFT spectral area

plots. We studied the autocorrelation sequences of the wavelet coefficient energies, probing for hidden deterministic oscillatory behavior and testing the degree of randomness in the transformed HRV signal.

We used also return maps (embedding dimension  $m = 2$ ) in order to identify the presence of possible underlying chaotic drives in the sympathetic and parasympathetic control systems through low- and high-frequency HRV modulations. The optimal return map lags were determined by nearest maxima of the autocorrelation function for the LF, HF, LF/HF and LF+HF octave band energies. Note, that the starting point of these lag searches was moved beyond an initial blanking period to avoid the first peak of the autocorrelation sequence.

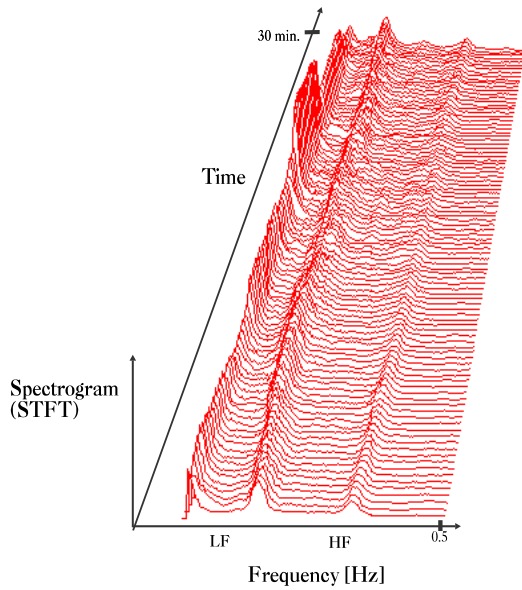
## Results

The standard analysis of the resampled long-term HRV series through the Short-Time Fourier Transform (0.0039-0.5 Hz, Hamming window) showed normal quasi-stationary frequency distributions in the LF and HF regions for normal subjects (Fig.3a) and various deviations in pathological recordings (Fig.3b).

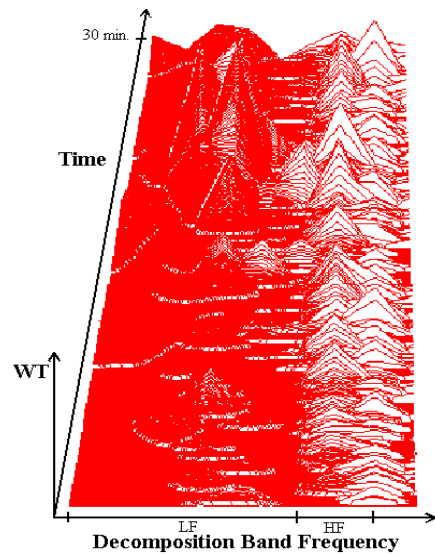
The dynamic multiresolution decomposition of the same HRV time series through the orthogonal wavelet transform showed different patterns, but the discrimination between normal (Fig.4a) and distorted (Fig.4b) cardiac rhythms was clearly observable in most cases. At the same time the DWT robustly included in the scalogram distinct characteristics of the transient events - their localization, duration and frequency components, which were almost ignored by the STFT spectra.

The autocorrelation functions of the LF (Fig. 6) and HF energy time series proved quite interesting, displaying additional modulation patterns in heart rate over several ranges of time - slow, but powerful fluctuations in the low frequency (LF) region of the heart rate variability and faster, but constantly decaying ones at high frequencies (HF) by normal subjects.

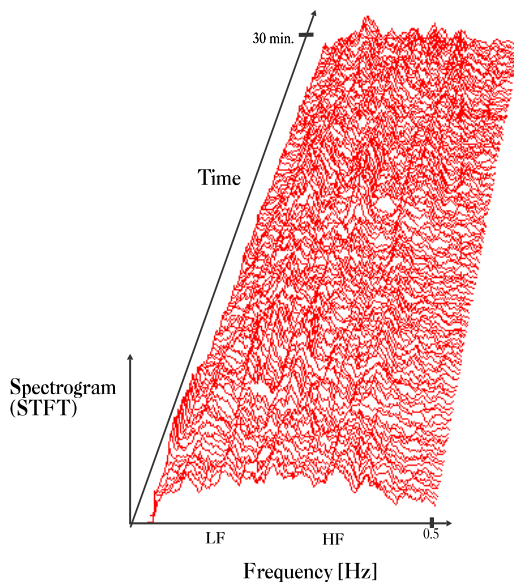
The search for non-deterministic patterns in the LF and HF band energies through return maps confirmed the different nature of LF and HF fluctuations in the heart rate. In case of a unitary fixed lag  $L=1$ , the LF energy of the wavelet decomposition for normal subjects preserved the known comet-shape cloud in phase space, manifesting increased variability for higher values and compact space



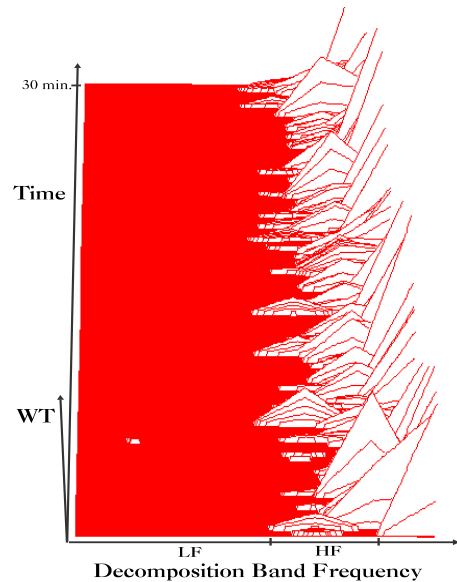
(a)



(a)



(b)



(b)

Fig.3. Short-Time Fourier Transform (compressed spectral array) of the R-R interval variations

(a) normal subject; (b) pathological case.

Each horizontal curve reflects the stationary frequency distributions of a single data window. (Frequency range: 0.0039-0.5 Hz, Total time range: 30 min, Time range of a single window: 256 s, Movement step: 16 s, Data Windowing: Hamming window). X-axis: Frequency, increasing from left to right side; Y-axis: Time, increasing from front to back; Z-axis: STFT magnitudes.

Spectral region definitions:

LF: 0.0039 - 0.12 Hz,

HF: 0.12 - 0.5 Hz.

Fig.4. The square moduli of the orthogonal wavelet transform coefficients (scalogram) for a long-term heart rate variability signal

(a) normal subject; (b) pathological case.

(Daubechies 4-tap wavelet, 3% denoising threshold)

Physiologically relevant frequency bands used:

LF: 0.00488 - 0.00976 Hz, 0.00976 - 0.01953 Hz,  
 0.01953 - 0.03906 Hz, 0.03906 - 0.07812 Hz,  
 0.07812 - 0.15625 Hz;

HF: 0.15625 - 0.3125 Hz, 0.3125 - 0.625 Hz.

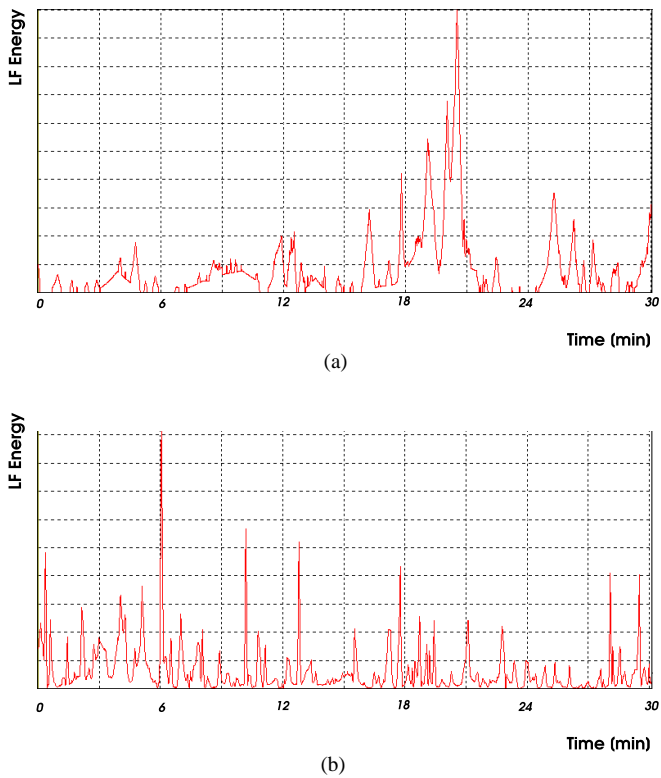


Fig.5. LF octave bands energy plot (normalized)

(a) normal subject; (b) pathological case.

Parasympathetic activity is regarded as more correlated with high frequency (HF) components of spectra, but both sympathetic and parasympathetic tonuses (mostly sympathetic) seem to determine the low frequency (LF) components of the HRV [3].

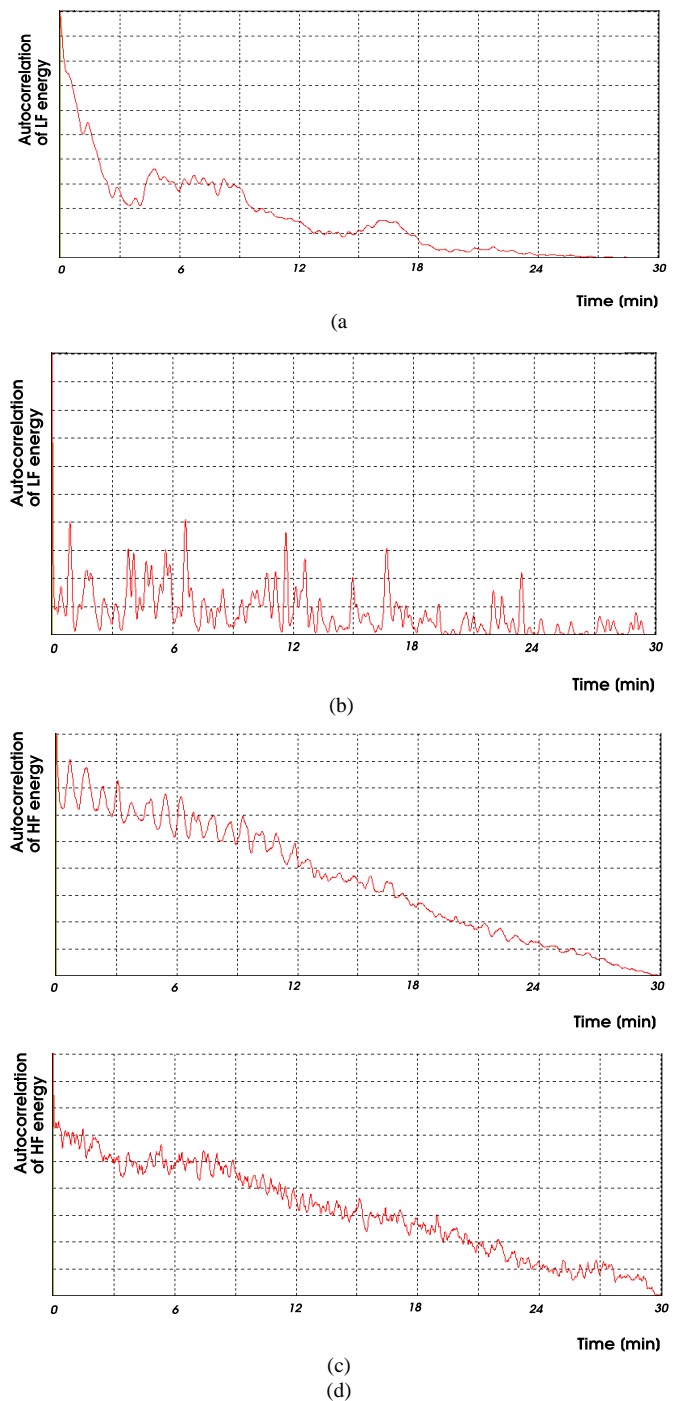


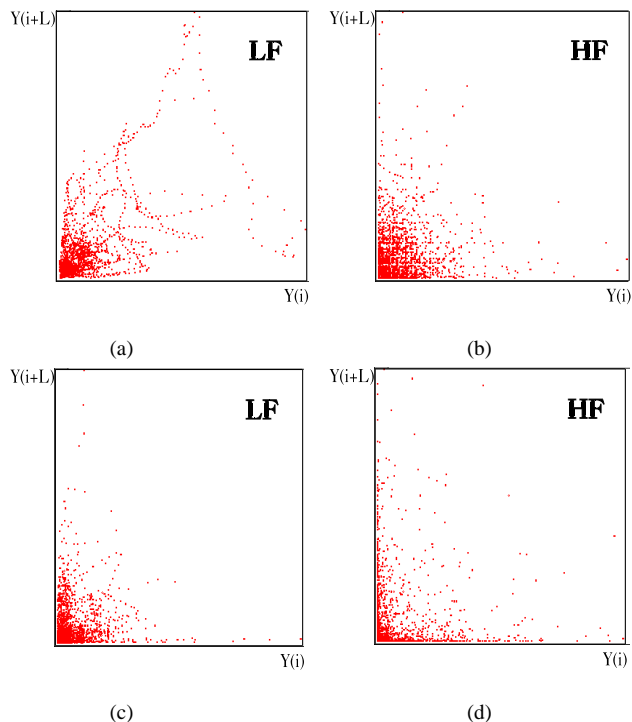
Fig.6. Autocorrelation function (normalized) of the LF energy of the multiresolution wavelet transform for identification of oscillatory repetitive patterns in the heart rate variability over several ranges of time.

(a) autocorrelation of LF energy for a normal subject, indicating slow, but powerful fluctuations in the low frequency region of the HRV;

(b) autocorrelation of LF energy in case of heart rhythm disorder;

(c) autocorrelation of HF energy for the same normal subject at high frequencies of the HRV, constantly decaying in time, but modulated with fast oscillations;

(d) autocorrelation of HF energy in the same pathological case.



*Fig.7. Normalized return maps for identification of strange-attractor-driven and repetitive patterns*

*(a) LF energy return map of a normal subject, indicating possible underlying chaotic regulation in the autonomic innervation and functional control processes; (b) HF energy return map, reflecting a regular modulation pattern in the normal subject; (c) LF energy return map of a subject with pathological HRV; . (d) HF energy return map of the pathological case with no regular modulation patterns in phase space.*

*Lags are derived from autocorrelation maxima.*

localization for smaller R-R intervals. The HF region maintained its scattered shape under all lag values tested and the return map lags influenced mostly the presence of row-column-like regularity structures in the cloud.

When the lag was defined adaptively from the autocorrelation maxima, LF return maps for normal subjects (Fig. 7a) showed chaotic-like patterns, while their HF counterparts (Fig. 7b) were higher structured than in the case of the unitary lag described earlier. Pathological cases showed distinct pattern deviations in the phase space - in the LF (Fig. 7c), as well as in the HF (Fig. 7d) regions.

Discrimination tests between normal and pathological HRV patterns were carried out visually, although exact

methods have to be applied for numerical evaluation in further studies.

Due to intrinsic time resolution (increasing with the frequency) properties of the DWT we found more long-term dynamical changes in the HRV (especially in the LF), than conventionally sensed by methods like STFT or AR-models.

We were also able to study in detail the very low frequency (VLF) range with the help of the multiresolution wavelet decomposition. The finest frequency range our data lengths allowed us to investigate was 0.000305-0.00061 Hz, which made possible the detection and fine temporal localization of postural and physical factors in the HRV signal.

### Limitations

Choice of the most appropriate wavelet basis is a matter of great importance in wavelet applications. A trade-off between desired properties should be considered, accounting for orthogonality, compact support, symmetry, number of vanishing moments, smoothness and rationality of filter coefficients. Daubechies [8] constructed the only wavelets, except for the Haar wavelet, that are orthogonal and have compact support on an interval of length  $2N-1$ . Unfortunately they are not symmetric, which leads to phase distortions. As for the Haar wavelet ( $N=1$ ), it is more or less inappropriate for HRV analysis. One possible solution to the problem of asymmetry in Daubechies wavelets is the application of biorthogonal wavelets, whose decomposition quadrature mirror filters are no longer orthogonal to each other, but only to their complementary reconstruction filter pair. Smoothness is another property of wavelets, that is subject of compromise between compact support and frequency localization of the filters. Coding performance and errors due to setting small wavelet coefficients to zero are dependent on the filter length. However, since signal compression is not a primary goal for HRV analysis, obtaining minimal amount of data for decision making after DWT makes orthogonality and compact support important factors by the choice of wavelets. In future studies better physiological interpretation of the HRV multiresolution decomposition should enable us to apply decision-making best basis selection algorithms for further optimizations.

Since the energy of the wavelet coefficients is a bilinear function of the HRV signal, cross-terms may interfere in the evaluated time-scale patterns of the scalogram. In such case phase information or other supplementary analysis tools

could help prevent misinterpretation and improve the performance.

## Conclusion

A major distinction of the DWT representation, which could eventually change the interpretation of HRV, is that this method inherently accounts for frequency events in time. Constant-Q properties of the DWT allow us to study modulation patterns in even very long-term signals over all physiologically meaningful scales. Discrimination between stable, metastable and unstable HRV components is essential for the study of their physiological origin and influence. It is important to consider the degree of frequency-time spottiness and repetition patterns through various methods. Although statistical measures give a fair common picture, we observed the presence of strange attractor-like patterns through the return map plots -- especially of the LF octave bands of the DWT. This could be possibly explained by non-deterministic control processes in the underlying thermal, vasomotor and hormonal regulation and in the specific interaction patterns of the sympathetic and parasympathetic nervous systems. On the other hand, the structured HF return maps of normal subjects corresponded to a more regular respiratory control modulation, which was absent in pathological cases.

We studied as well some other condensed characterization parameters like mean and median scale representations, skewness and variance, applied for the entire studied time range. They might be considered useful for fast screening purposes, although they are not directly comparable to conventional STFT and AR spectra and may be somewhat difficult to evaluate.

These results suggest that the multiresolution orthogonal wavelet decomposition appears to be a useful technique for characterization of the dynamic changes and repetition patterns of the various HRV signal components. It can be also advantageous in fast risk analysis evaluations and in extraction of fiducial markers for miscellaneous autonomic function events.

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